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AN OBVIOUS ISOSPIN BREAKING CORRECTION TO ϵ' OF KINEMATICAL ORIGIN

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Abstract

Isospin breaking correction to the $\Delta I = \frac{1}{2}$ decay of $K \to \pi\pi$ generates a large enough contribution to ϵ' through an induced $\Delta I = \frac{3}{2}$ amplitude in K_L decay. Aside from the π-η and π-η' mixing contributions, there is a correction of kinematical origin due to the final-state $\pi^{\pm}$-π^0 mass difference, which is unambiguously calculable from the low-energy off-shell behavior of the $K \to \pi\pi$ amplitude. This correction to ϵ' reduces the isospin breaking parameter Ω_{IB} by 0.06, which is nearly one half of the π-η mixing effect computed with chiral Lagrangians to $O(p^2)$.

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I. INTRODUCTION

The ϵ' parameter measures the CP phase difference between the $\Delta I = \frac{1}{2}$ and $\Delta I = \frac{3}{2}$ amplitudes in K^0 decay:

$$\epsilon' = \frac{-i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \omega \Big(\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0} - \frac{\mathrm{Im}A_2}{\mathrm{Re}A_2} \Big), \tag{1}$$

where $A_{0,2}$ are the $K \to \pi\pi$ amplitudes into $I_{\pi\pi} = 0$ and $I_{\pi\pi} = 2$. In the Standard Model with the conventional CP phase assignment to quark mixing, the penguin decay is practically the entire source of $\mathrm{Im}A_0$. Because of the $\Delta I = \frac{1}{2}$ enhancement, the isospin breaking correction to the penguin decay is a major $\Delta I = \frac{3}{2}$ contribution to ϵ'. The π-η and π-η' mixing contributions to the isospin breaking have been computed [1].

The isospin breaking effect is parametrized by

$$\Omega_{IB} = \frac{\mathrm{Im}A_2^{IB}}{\omega \mathrm{Im}A_0}, \tag{2}$$

where $\omega = \mathrm{Re}A_2/\mathrm{Re}A_0 \simeq 1/22$, and A_2^{IB} is the $\Delta = \frac{3}{2}$ amplitude induced by the electromagnetic or the u-d quark mass difference correction to the $\Delta I = \frac{1}{2}$ amplitude. The isospin breaking parameter Ω_{IB} enters ϵ' as

$$\epsilon' = \frac{-i}{\sqrt{2}} e^{i(\delta_2 - \delta_0)} \omega \Big[\frac{\mathrm{Im}A_0}{\mathrm{Re}A_0}(1 - \Omega_{IB}) - \frac{\mathrm{Im}A_2'}{\mathrm{Re}A_2} \Big], \tag{3}$$

where $\mathrm{Im}A_2' = \mathrm{Im}(A_2 - A_2^{IB})$. The contribution of the π-η mixing to Ω_{IB} is

$$\Omega_{IB}^{\pi\eta} = 0.13 \tag{4}$$

according to the calculation of $O(p^2)$ in chiral Lagrangian [1,2]. The recent calculation to $O(p^4)$ raised the value to 0.16 ± 0.03 [2]. After including π-η' mixing, $\Omega_{IB}^{\pi\eta+\pi\eta'} = 0.25 \pm 0.08$ has also been quoted [1].

The purpose of this short paper is to point out that there is one obvious isospin breaking correction to the $\Delta I = \frac{1}{2}$ amplitude which arises from the $\pi^{\pm}$-π^0 mass difference of the final pions. This is an effect of $O(p^2)$ in the momentum expansion and numerically as large as one half of Eq. (4) with the opposite sign. Considering its significance in testing the Standard Model with ϵ', we wish to call attention to this obvious correction.

II. ISOSPIN BREAKING BY EXTERNAL PION MASS DIFFERENCE

The isospin structure of the $K \to \pi\pi$ decay amplitudes is parametrized as

$$\begin{aligned} A(K^0 \to \pi^+\pi^-) &= A_0(p_K^2, p_{\pi^\pm}^2, p_{\pi^\pm}^2) + \frac{1}{\sqrt{2}} A_2(p_K^2, p_{\pi^\pm}^2, p_{\pi^\pm}^2), \\ A(K^0 \to \pi^0\pi^0) &= A_0(p_K^2, p_{\pi^0}^2, p_{\pi^0}^2) - \sqrt{2} A_2(p_K^2, p_{\pi^0}^2, p_{\pi^0}^2). \end{aligned} \tag{5}$$

The amplitude A_0 in Eq. (5) for $I = 0$ actually hides an $I = 2$ component through the external pion mass dependence. The external four-momentum dependence of the A_0 amplitude has been well known to $O(p^2)$:

$$A(p_K^2, p_a^2, p_b^2) = \frac{1}{2}A'(0)(2p_K^2 - p_a^2 - p_b^2) + O(p^4), \tag{6}$$

where $A'(0)$ is a constant, and p_K and $p_{a,b}$ denote momenta of K^0 and two final pions. This robust external momentum dependence is a consequence of SU(3) symmetry of strong interaction and charge conjugation property of the parity-violating nonleptonic decay interaction [3], though it is more often discussed with chiral symmetry nowadays. The $\pi^{\pm}$-π^0 mass difference of the final pions in Eq. (6) generates an effective $\Delta I = \frac{3}{2}$ amplitude and contributes to Ω_{IB} through $\mathrm{Im}A_2$. To our surprise, this correction has not been counted in literature.

On the mass shell the A_0 amplitude of Eq. (6) has the external mass dependence to $O(m_P^2)$,

$$\begin{aligned} A_0(K^0 \to \pi^+\pi^-) &= A'(0)(m_{K^0}^2 - m_{\pi^\pm}^2), \\ A_0(K^0 \to \pi^0\pi^0) &= A'(0)(m_{K^0}^2 - m_{\pi^0}^2). \end{aligned} \tag{7}$$

A $\Delta I = \frac{3}{2}$ amplitude emerges from the difference $m_{\pi^\pm}^2 - m_{\pi^0}^2$ in Eq. (7). Define the induced $\Delta I = \frac{3}{2}$ amplitude as

$$A_2^{IB} \equiv -\frac{\sqrt{2}}{3}A'(0)(m_{\pi^\pm}^2 - m_{\pi^0}^2). \tag{8}$$

Moving A_2^{IB} from A_0 to A_2, we can rewrite Eq. (5) up to the $O(p^4)$ correction as

$$\begin{aligned} A(K^0 \to \pi^+\pi^-) &= A'(0)(m_{K^0}^2 - \langle m_\pi^2 \rangle) + \frac{1}{\sqrt{2}}(A_2 + A_2^{IB}), \\ A(K^0 \to \pi^0\pi^0) &= A'(0)(m_{K^0}^2 - \langle m_\pi^2 \rangle) - \sqrt{2}(A_2 + A_2^{IB}), \end{aligned} \tag{9}$$

where $\langle m_\pi^2 \rangle = \frac{1}{3}(2m_{\pi^\pm}^2 + m_{\pi^0}^2)$.

Substituting the imaginary part of Eq. (8) in Eq. (2), we obtain the contribution of $\mathrm{Im}A_2^{IB}$ to Ω_{IB} as

$$\begin{aligned} \Delta\Omega_{IB} &= -\frac{\sqrt{2}}{3\omega}\Big(\frac{m_{\pi^\pm}^2 - m_{\pi^0}^2}{m_{K^0} - \langle m_\pi^2 \rangle}\Big), \\ &= -0.058. \end{aligned} \tag{10}$$

This is the external mass difference contribution to Ω_{IB} to $O(p^2)$ of A_0. The sign is opposite to the π-η and π-η' contributions, and the magnitude is nearly one half of the $O(p^2)$ contribution of π-η mixing [1,2],

$$\begin{aligned} \Omega_{IB}^{\pi\eta} &= \frac{m_d - m_u}{3\sqrt{2}(m_s - m_{u,d})}, \\ &= 0.13, \end{aligned} \tag{11}$$

and more than a third of $\Omega_{IB}^{\pi\eta} = 0.16 \pm 0.03$ [2] which includes $O(p^4)$.

III. DISCUSSION

It is obvious that this pion mass difference contribution is not counted in the π-η and π-η' mixing calculation. It is purely kinematical in origin. Our number is at the level of $O(p^2)$ in the language of chiral Lagrangian expansion. While the π-η mixing correction is fairly clean to $O(p^2)$, the $O(p^4)$ correction contains more dynamical uncertainties. In our calculation we have ignored an explicit SU(3) breaking in $A_0(p_K^2, p_a^2, p_b^2)$ of Eq. (6). This is the only possible source of uncertainty involved in Eq. (10). We make a remark on it.

The s-u/d quark mass difference in internal lines generates an SU(3)-breaking A_0 term that does not vanish in the soft meson limit. In chiral Lagrangians, one quark mass insertion does not generate a nonderivative term for the $\Delta I = \frac{1}{2}$ decay since $\mathrm{tr}(\lambda_6 M_q U^\dagger)$ can be diagonalized away. Therefore there is no correction of $O(m_P^2)$ to our result. The internal quark mass correction is of $O(m_P^2) \times O(p^2)$, which is the same as $O(p^4)$ on the mass shell of $K \to \pi\pi$. An explicit computation of $O(p^4)$ to one-loop was made with the kaon off mass shell, while keeping the pion masses on shell and degenerate [4]. In this calculation the explicit SU(3) breaking of the quark mass insertion manifests itself in the terms proportional to $m_K^2 - m_\pi^2$ instead of $p_K^2 - m_\pi^2$. This SU(3) breaking turns out to be very small primarily because of the loop factor $1/(4\pi f_\pi)^2$. Therefore the next-order correction to Eq. (10) is expected to be small. If one wishes to compare our result with the $O(p^4)$ result of $\Omega_{IB}(= 0.16 \pm 0.03)$ from chiral Lagrangians, one had better compute for consistency the A_0 amplitude with the pions off mass shell and include the next order terms of $(m_K^2 - m_\pi^2) \times O(p^2)$ in Eq. (10). Though it is small in magnitude, the next-order correction to Eq. (10) contains dynamical uncertainty.

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